Fitting CMB data with cosmic strings and inflation

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(Dated: 8 February 2007)

We perform a multi-parameter likelihood analysis to compare measurements of the cosmic microwave background (CMB) power spectra with predictions from models involving cosmic strings. We explore the addition of strings to the inflationary concordance model, involving an adiabatic primordial power spectrum with a power-law tilt n_s, as well as the Harrison-Zeldovich (HZ) case $n_s = 1$. Using ACBAR, BOOMERANG, CBI, VSA and WMAP data we show that of the models investigated, the HZ case with strings provides the best fit to the data relative to the freedom in the model, having a moderately higher Bayesian evidence than the concordance model. For HZ plus strings, CMB data then implies a 10 ± 3 % string contribution to the temperature power spectrum at multipole $\ell = 10$. However, with non-CMB data included, finite tilt and finite strings are approximately on par with each other. Considering variable n_s, we then find a 95% upper limit of the string fraction of 11%, corresponding to $G\mu < 0.7 \times 10^{-6}$ (where G is Newton's constant and μ is the string tension).

arXiv:astro-ph/0702223v1 8 Feb 2007

The inflationary paradigm is successful in providing a match to measurements of the cosmic microwave background (CMB) radiation and it appears that any successful theory of high energy physics must be able to incorporate inflation. While ad-hoc single-field inflation models can provide a match to the data, more theoretically motivated ones commonly predict the existence of cosmic strings [1]. These strings are prevalent in supersymmetric D- and F-term hybrid inflation models (see [2] for a review) as well as occurring frequently in grand-unified theories (GUTS) [3], while string theory can also give rise to strings of cosmic extent [4]. Hence their observational consequences are important, including their sourcing of additional anisotropies in the CMB radiation.

In this letter we present a multi-parameter fit to CMB data of models involving cosmic strings. It is the first such analysis to use CMB predictions for (local) cosmic strings from field theory simulations, which we describe in [5]. Using these we show that a finite contribution from strings is moderately favored by current CMB data, and we discuss the statistical significance of this preference via Bayesian evidence [6]. We then consider the concordance between the values of the cosmological parameters implied by the string-included fits and those values determined using non-CMB approaches; before additionally including these non-CMB data in our calculations.

In the combined inflation plus strings case, inflation creates primordial matter and radiation perturbations that evolve passively until today, during which time cosmic strings actively source additional perturbations. Given the small size of the observed CMB anisotropies, the perturbations may be treated linearly and any coupling between those seeded by the two mechanisms can be ignored. The string and inflation perturbations can therefore be evolved via separate calculations, yielding two contributions to the CMB power spectrum that are statistically independent. Therefore these are simply added together to give the total power spectrum.

However, calculating the cosmic string component is still challenging and all previous comparisons of the total power spectrum against data have relied upon one of the following simplifying models to give the string contribution. The width of (local) strings is very much smaller than their separation at times of importance for CMB calculations, making their resolution in cosmic simulations very difficult. Therefore the calculations in [7] involved representing the strings as connected 1D line segments. These were then evolved according to the Nambu-Goto equations of motion, appropriate in the zero-width limit, but the representation means that the radiative decay of the strings has to be modeled in an ad-hoc manner. A further simplification is to represent the strings as unconnected segments, with stochastic velocities, and to randomly remove them in order to model decay. This simpler method gives results in broad agreement with [7] and has enabled the CMB parameter fitting of [8, 9]. On the other hand, the third approach is to simulate instead global strings, which do not localize their energy into the string cores. They may hence be left unresolved and can be tackled more easily in field-based simulations, allowing the CMB calculations of [10], which were used in [11, 12].

In contrast, here we use our results from [5], where we employed the Abelian Higgs model to make CMB calculations for local U(1) strings that naturally included radiative decay. Being local strings, their width had to be resolved and computational limitations therefore restricted the simulations to early times, when the string separation was ~ 100 times their radius. However, strings are believed to evolve toward a scaling regime such that their statistical properties are a function of a single scale: the horizon size. This assumption, which was carefully checked using the simulations themselves, enables the statistical results to be applied to the later times required in CMB calculations. Hence we believe these to

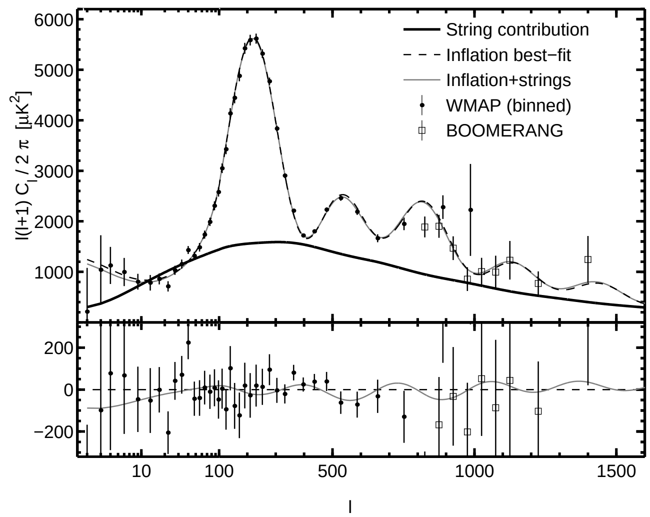

FIG. 1: The temperature power spectrum contribution from cosmic strings, normalized to match the WMAP data at $\ell = 10$, as well as the best-fit cases from inflation only (model PL) and inflation plus strings (PL+S). These are compared to the WMAP and BOOMERANG data. The lower plot is a repeat but with the best-fit PL case subtracted, highlighting the deviations between the predictions and the data.

be the most accurate such calculations to date.

Our result for the temperature power spectrum contribution [5] is shown in Fig. 1, where it is compared to both observational data and the best-fit inflationary model. The normalization of the inflation and string power spectra components are free parameters, with that for strings being proportional to $(G\mu)^2$ (where μ is the energy per unit length and G is the gravitational constant). For Fig. 1 the normalization of the string component has been set to match the data at multipole $\ell = 10$, corresponding to $G\mu = (2.0 \pm 0.2) \times 10^{-6}$, a factor of 2-3 higher than the corresponding value from previous work [7, 8, 13]. Clearly a string component this large is ruled out and we hence introduce the parameter f_{10}, which is the fractional contribution from cosmic strings to the temperature power spectrum at $\ell = 10$.

While the primary CMB measurements are of its temperature anisotropies, additional information can be provided by variations of its polarization across the sky. Only the so-called E-mode polarization has been detected at present, to which strings contribute weakly, however we still include the string contribution here for completeness. A discussion of our polarization results from [5], including the B-mode (to which strings may contribute strongly), is presented in [14].

For inflation plus global defect scenarios, we have previously shown that the freedom for the cosmological parameters to vary enables the inflationary contribution to change in order to accommodate a large defect component and still fit CMB data [15]. Here we therefore follow the method of [15] and employ a full multi-parameter

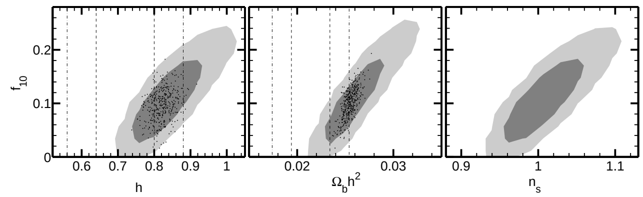

FIG. 2: The 2D marginalized likelihood distributions from CMB data (only) for f_{10} versus h, $\Omega_b h^2$, and n_s. Contours show the 68 and 95% confidence regions for model PL+S while the 400 MCMC points indicate the prefered region when $n_s = 1$. The vertical lines on the h and $\Omega_b h^2$ plots show the 68 and 95% confidence limits from the HKP and BBN measurements.

Markov chain Monte Carlo (MCMC) likelihood analysis for the cosmic string case. While recalculating the inflationary component at a different cosmology takes a few seconds, for the string contribution this takes many hours, which appears to render a full MCMC analysis unfeasible. Therefore following our previous work, we fix the form of the string component and vary only its normalization via $G\mu$. Given that strings are sub-dominant, this amounts to a small error in the total inflation plus strings prediction and, if it is well below the uncertainties in the CMB data [24], the results will not be noticeably affected. We hence use a version of the standard MCMC code, `CosmoMC` [16], that we have modified to include the fixed-form cosmic string component.

We primarily consider four different models: two parameterizations of the primordial power spectrum, both with and without strings. We always allow for variations in the Hubble parameter h; the physical baryon and total matter densities $\Omega_b h^2$ and $\Omega_m h^2$; and the optical depth to last scattering τ. We then either take Harrison-Zeldovich (scale-invariant) adiabatic primordial perturbations with amplitude A_s or add the additional freedom of a power-law tilt n_s: $A_s^2 \rightarrow A_s^2 (k/k_0)^{n_s}$. This yields the two zero-string models which we label as HZ and PL respectively, with PL being the established inflationary concordance model and HZ being a restriction of this: $n_s = 1$. We add strings to these two models yielding models HZ+S and PL+S, which therefore have the extra parameter $(G\mu)^2$. Then, in the later stages of our discussion, we also consider primordial tensor perturbations and a finite running of the scalar spectral index $dn_s/d\ln k$ but we will assume negligible neutrino mass and flat space throughout.

The results when using CMB data from the ACBAR, BOOMERANG, CBI, VSA and WMAP projects [17] are illustrated in Fig. 2. This shows the marginalized 2D likelihood surfaces for f_{10} versus h, $\Omega_b h^2$, and n_s for both HZ+S (points) and PL+S (contours). In the PL+S, there is a significant degeneracy, involving primarily these parameters, which allows large values of f_{10} to fit the data. It also allows large ranges for h and $\Omega_b h^2$, but recalculating the string component at the parameters required for

model ID	no. param.	CMB only		CMB+HKP+BBN	
		$\Delta\chi^2_{\rm eff}$	evidence	$\Delta\chi^2_{\rm eff}$	evidence
HZ	5	$+7.7$	0.35 ± 0.03	$+10$	0.133 ± 0.005
PL	6	0	1	0	1
HZ+S	6	-3.9	7.3 ± 1.2	$+0.9$	0.76 ± 0.13
PL+S	7	-3.9	1.2 ± 0.1	-1.6	0.19 ± 0.01

TABLE I: The $\Delta\chi^2_{\rm eff}$ and relative Bayesian evidence values for the examined models using the CMB, HKP and BBN data.

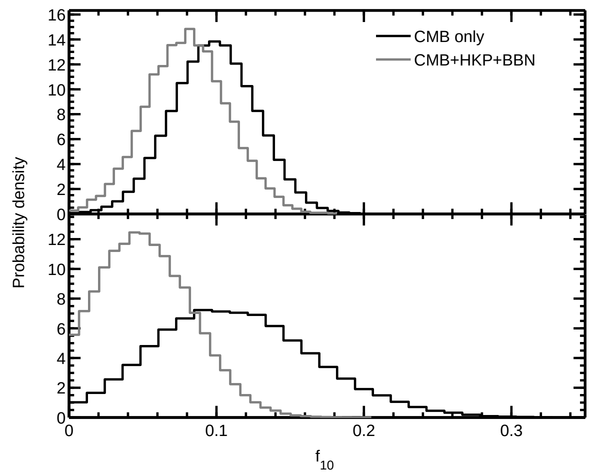

FIG. 3: The 1D marginalized likelihood for the fractional contribution of strings to the temperature power spectrum at $\ell = 10$ in the HZ+S model (upper) and PL+S model (lower) using CMB data alone and also with non-CMB data included.

high and low f_{10} yielded an insignificant change, which confirmed validity of our fixed-form approach.

The degeneracy is further explored in Fig. 1 (lower), which shows the deviations between the best-fit PL+S and PL cases. It can hence been seen that not only does the shown PL+S case of $f_{10} = 0.099$ fit the data, the fit is marginally improved over the zero-string case (see endnote [25] for the best-fit parameter values). Indeed, when the maximum likelihood values $\mathcal{L}$ are compared via: $\Delta\chi^2_{\rm eff} = -2\ln(\mathcal{L}_{\rm PL+S}/\mathcal{L}_{\rm PL})$, we obtain $\Delta\chi^2_{\rm eff} = -3.9$ at the expense of a single extra parameter. However, the PL+S best-fit value of $n_{\rm s}$ is extremely close to unity and hence HZ+S has an almost identical $\mathcal{L}$ value. Therefore model HZ+S gives $\Delta\chi^2_{\rm eff} = -3.9$ relative to the concordance model and yet has the same number of parameters.

A more complete analysis of the freedom in a model is provided by its Bayesian evidence value [6]. We calculate evidence ratios between models using the Savage-Dickey method [18, 19] with flat priors of $0 < f_{10} < 1$ and $0.75 < n_{\rm s} < 1.25$, giving the results shown in the table. While the relative evidence of PL+S to PL under CMB data is barely distinguishable from unity, model HZ+S is moderately preferred relative to PL with a Bayes factor of 7.3 ± 1.2. Hence the data favors cosmic strings over a tilted primordial power spectrum.

However, the points plotted in Fig. 2 indicate that HZ+S predicts values of h and $\Omega_{\rm b}h^2$ that are larger than those from non-CMB measurements. The Hubble Key Project (HKP) yielded $h = 0.72 \pm 0.08$ [20] while the measurement of deuterium abundance in high redshift gas clouds, combined with big bang nucleosynthesis (BBN) calculations, gives $\Omega_{\rm b}h^2 = 0.0214 \pm 0.0020$ [21]. Both of these are lower than the corresponding HZ+S predictions. This is a not a problem with model PL, which yields excellent concordance with these independent data, and it is only the very large f_{10} values in PL+S that are at odds with them. Although there remains some questions over the BBN constraints on $\Omega_{\rm b}h^2$, with BBN calculations using different isotopes not yielding results that are wholly consistent [21], it is interesting to investigate the effect of adding these data. When incorporating them into the MCMC procedure (with Gaussian likelihoods), it is the BBN that yields the strongest constraint (see Fig. 2) and we will not discuss results of adding the data separately. The changes in the marginalized 1D likelihood for f_{10} under HZ+S and PL+S are shown in Fig. 3. With the shown reduction in the allowed f_{10} values, model PL+S no longer favors $n_{\rm s} = 1$ (giving $n_{\rm s} = 0.97 \pm 0.02$) and unsurprisingly, the Bayes factor for HZ+S relative to PL is now reduced such that the preference for the string model is removed.

Hence while CMB data under model HZ+S gives a plausible detection of cosmic strings with $f_{10} = 0.10 \pm 0.03$ (or $G\mu = [0.65 \pm 0.10] \times 10^{-6}$), if the BBN result is believed then this model suffers a moderate concordance problem. Further, tilt is preferred in the zero-string case so it would not be fair to claim a 3-σ detection from CMB data without considering the impact of $n_{\rm s} \neq 1$. In the PL+S case, the f_{10} distribution gives $f_{10} = 0.11^{+0.05}_{-0.06}$, showing that the central value is stable to the addition of tilt but the uncertainty estimate is sensitive to the extra freedom. Then, when HKP and BBN data are added, this model does yields only a 1-σ preference for strings and we therefore quote (95% confidence) upper bounds for f_{10} under PL+S. These are $f_{10} < 0.21$ (CMB) and $f_{10} < 0.11$ (CMB+HKP+BBN) corresponding to $G\mu < 0.9 \times 10^{-6}$ and $G\mu < 0.7 \times 10^{-6}$ respectively. Differences relative to the $G\mu$ limits quoted by other authors are largely due to our use of a more complete string model.

When the freedom for either a finite primordial tensor contribution or a running $n_{\rm s}$ was incorporated, we find results as follows. As a potential addition to model PL, tensor modes give a negligible (and possibly zero) improvement to the fit. Their addition to PL+S is more favorable, slightly increasing the required $n_{\rm s}$ values (as well as the those for h and $\Omega_{\rm b}h^2$), but the above numerical results are not affected enough to warrant further discussion here. Adding finite $dn_{\rm s}/d\ln k$ to model PL does give a marginal improvement to the fit, with CMB data preferring a slight negative running. This lowers small and large scales relative to intermediate ones and may hence be thought to have a similar effect as strings. However, adding strings smooths out the acoustic peaks and in fact there is little correlation between $dn_{\rm s}/d\ln k$ and f_{10}. Hence when adding running to PL+S, we find merely a marginal improvement in the fit and an insignificant change in the above upper limits.

We have not attempted to apply further constraint to the models via galaxy survey data and the linearized matter spectrum. This is because, unlike inflationary perturbations, those from strings are expected to be highly non-Gaussian. The nature of the statistics was shown in [22] to affect the redshift of reionization, which is of course dependent upon the growth of structure. We believe a more complete understanding of structure formation in inflation plus string scenarios is required before galaxy surveys can be used to make reliable inferences about the matter power spectrum in such cases. Similarly, any bounds from cosmic rays [23] or gravitational wave production by string loops [9] are sensitive to the small-scale network properties, which are poorly understood, whereas the CMB is sensitive to the more easily simulated large-scale properties.

While we have shown that $n_{\rm s}=1$ gives a good fit to the CMB data when strings are included, strings also remove the pressure on $n_{\rm s} \gtrsim 1$ that otherwise exists under model PL: $n_{\rm s} = 0.95 \pm 0.02$. As has been shown elsewhere [9] (using unconnected segments to model strings) this is potentially important for supersymmetric hybrid inflation models. These predict $n_{\rm s} > 0.98$ but negligible tensor components and are disfavored by CMB data without strings considered. However, here we also find that cases giving $n_{\rm s} > 1$ are still disfavored once non-CMB data is incorporated: $n_{\rm s} = 0.97 \pm 0.02$.

Finally, we note that our bounds have been derived only for classical Abelian Higgs strings with equal vector and scalar particle masses [5], and that, for example, D-term inflation may be more accurately treated using simulations with different values. Similarly, changes in the CMB predictions for strings may be found with D- and F-string composites from string theory [4], or with other model variations. Hence if a confirmed string detection is made in the future, it may be possible to learn a great deal about inflation and high energy physics.

We would like to thank Rob Crittenden, Richard Battye, Andrew Liddle, and David Parkinson for helpful conversations. We acknowledge financial support from PPARC (N.B., M.H), the Swiss NSF (M.K.), the US NSF, Marie Curie Intra-European Fellowship MEIF-CT-2005-009628 and FPA2005-04823 (J.U.).

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